Digital Twin-Assisted Performance Prediction of FRP-Confined Concrete Structures Under Variable Loading Conditions

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Abstract

Fiber-reinforced polymer (FRP) confinement is an effective method for enhancing the axial capacity and ductility of concrete columns, yet performance prediction under variable loading remains difficult because confinement mechanics, cumulative damage, and measurement uncertainty interact across time. This paper develops a digital twin-assisted methodological framework for predicting the performance of FRP-confined concrete structures under variable loading conditions. The framework combines a mechanics-based virtual model of confinement with a state-space representation of damage evolution, a Gaussian-process surrogate for fast prediction, and Bayesian calibration for updating uncertain parameters as monitoring data become available. Load-path descriptors are used to represent variable amplitude and cyclic effects that are not captured by monotonic confinement models alone. The central contribution is an integrated prediction architecture that separates physical state variables from surrogate model parameters, propagates uncertainty explicitly, and enforces consistency between confinement mechanics and data-driven correction. The paper derives the governing relationships for confining pressure, state transition, measurement, damage accumulation, residual capacity, and Bayesian updating, and discusses identifiability, model discrepancy, calibration design, and decision implications. The framework provides a technically defensible basis for condition-aware assessment of FRP-confined structures and clarifies where empirical calibration is essential before operational deployment.

Keywords

digital twin; FRP-confined concrete; variable loading; Bayesian calibration; Gaussian process; structural performance prediction

1. Introduction

FRP-confined concrete derives its improved axial behavior from lateral restraint provided by a composite jacket. The jacket restricts dilatation, increases peak stress, and can

substantially improve post-peak ductility, but the resulting stress-strain response depends on jacket stiffness, concrete grade, specimen geometry, and failure mode. Design-oriented confinement models have therefore been developed to relate lateral confining pressure to enhanced strength and strain capacity [1]. Earlier confined-concrete formulations provide the broader mechanical basis for representing lateral restraint, peak strength, and post-peak degradation [2], while structural applications require these relationships to be embedded in member-level assessment procedures [3]. In practice, however, confinement models are usually calibrated for monotonic loading and controlled laboratory conditions, whereas many structures experience variable loading caused by traffic, wind, seismic excitation, temperature cycles, or construction sequences.

Experimental and database studies have clarified important sources of variability in FRP-confined concrete. Test databases have been used to identify the influence of unconfined concrete strength, jacket stiffness, fiber orientation, and rupture strain on axial capacity and ultimate strain [4]. Confinement models for high-strength concrete require separate treatment because brittleness and dilatation differ from those of normal-strength concrete [5]. Cyclic compression introduces additional degradation mechanisms, including stiffness deterioration, residual strain accumulation, and hysteresis in the jacket-concrete interaction [6]. These effects mean that a single monotonic confinement curve is insufficient for performance prediction under variable loading, especially when the structure is monitored over time and decisions depend on remaining capacity rather than only initial strength.

Data-driven prediction has been applied to FRP-confined concrete with useful effect. Neural-network models have been used to estimate confined compressive strength from mixture, geometry, and jacket variables [7], and such models can capture nonlinear interactions that are cumbersome in closed-form design equations. However, purely empirical predictors do not generally represent internal damage state, load-path history, or measurement uncertainty, and they may extrapolate poorly outside the calibration domain. Digital twins offer a different organizing principle by coupling a virtual representation of a physical asset with data assimilation and prediction [8]. Digital-twin concepts have been extended to prognostics and health management [9] and to modeling workflows that combine simulation, sensing, and decision support [10]. For structural concrete, the digital twin should not be treated as a visual replica; it should be a calibrated, uncertainty-aware computational model whose state is updated as the asset responds to loading.

Predictive digital twins require a probabilistic formulation so that model parameters and unobserved states can be inferred from incomplete measurements [11]. The distinction between a digital model, a digital shadow, and a digital twin is important because only the latter involves automated or semi-automated information flow from the physical asset to the virtual model [12]. For FRP-confined concrete, this distinction has practical consequences: a jacket-level confinement model may provide a digital model, but a performance-prediction twin must also represent evolving damage, load history, and sensor-derived evidence. The methodological problem addressed here is how to construct such a twin when the underlying mechanical relationships are nonlinear, the available monitoring data are sparse, and the loading is variable.

This paper makes a methodological contribution rather than reporting a new test program. It formulates a digital twin architecture that integrates FRP confinement mechanics, variable-load state variables, Gaussian-process surrogate prediction, and Bayesian calibration. The framework is designed to preserve physical interpretability while allowing data-driven

correction, and it provides explicit equations for confining pressure, state transition, observation, damage accumulation, residual capacity, and uncertainty propagation. The intended outcome is not a claim of field-validated accuracy but a rigorous basis for prediction, calibration design, and technical assessment of FRP-confined concrete structures under changing load conditions.

2. Problem Formulation and Performance States

2.1 Confinement mechanics and virtual model

The virtual model represents the FRP jacket as a passive confining device whose lateral pressure depends on jacket thickness, elastic modulus, and hoop strain. For a circular section, the lateral confining pressure can be expressed as

$$f_l = 2t_f E_f \frac{\varepsilon_l}{D}$$

where f_l is the lateral confining pressure, t_f is the total jacket thickness, E_f is the effective elastic modulus of the FRP in the hoop direction, ε_l is the hoop strain, and D is the confined concrete diameter. This relationship is a standard thin-shell approximation and provides the first mechanical link between jacket deformation and concrete restraint [1]. It is appropriate for a digital twin because hoop strain can, in principle, be inferred from surface measurements or from a calibrated deformation model, while t_f, E_f, and D are design or inspection variables. The approximation neglects jacket bending, adhesive compliance, and localized debonding, so these effects must be represented as model discrepancy or as additional uncertain parameters rather than absorbed silently into a black-box predictor.

The unconfined concrete response supplies the reference state, whereas confinement modifies strength, peak strain, and post-peak behavior. Rather than selecting a single closed-form stress-strain law, the twin can represent a family of admissible confinement responses parameterized by f_l, unconfined strength, and ultimate strain. This is consistent with database-derived confinement models that relate normalized strength and strain enhancement to the confinement ratio [4]. For high-strength concrete, the same functional form may require altered limits and degradation parameters [5]. The virtual model therefore acts as a constrained prior: it defines physically admissible stress-strain trajectories, but it allows calibration parameters to adjust within ranges that remain consistent with confinement mechanics.

2.2 Variable loading and state evolution

Variable loading requires the twin to represent history-dependent response. A scalar capacity value cannot capture stiffness degradation, residual strain, or incomplete recovery after unloading. The state vector is therefore defined as x_k = [ε_c,k, f_l,k, D_k, κ_k]^T, where ε_c,k is the concrete axial strain at load step k, f_l,k is the current confining pressure, D_k is a normalized damage variable, and κ_k collects history descriptors such as maximum prior strain, cyclic amplitude, and unloading stiffness. The state evolves according to

$$x_{k+1} = F(x_k, u_k, \theta) + w_k$$

where u_k is the loading increment, θ is a vector of mechanical parameters, F is the transition operator defined by the confinement and damage laws, and w_k is process noise representing unmodeled mechanics and environmental variation. The transition operator is not a fitted empirical mapping in this formulation; it is a structured model in which confinement

mechanics constrain the admissible evolution of f_l,k and ε_c,k, while damage and history variables account for irreversible effects.

The observation model connects the latent state to available measurements. Let y_k denote measured quantities such as axial displacement, surface strain, acoustic emission features, or jacket hoop strain. The measurement equation is

$$y_k = h(x_k, \theta) + v_k$$

where h is the measurement operator and v_k is measurement noise. The operator may be nonlinear because strain gauges provide local measurements that are not identical to section-average strain, and acoustic emission features respond to cracking rather than directly to stress. Treating these measurements through an explicit observation model is preferable to directly regressing capacity on sensor readings, because it preserves the distinction between physical state and measurement artifact.

2.3 Prediction target under variable loading

The digital twin prediction target is a time-indexed performance distribution, not a single deterministic strength. At load step k, the relevant output may be current axial stress, residual stiffness, ultimate capacity, or probability of exceeding a serviceability or safety threshold. These outputs are functions of the latent state and uncertain parameters. The prediction problem can be stated as estimating p(R_k | Y_{1:k}), where R_k denotes a performance variable and Y_{1:k} is the measurement history. This formulation accommodates variable loading because the conditioning set includes the sequence of load increments and observed responses, not merely the current load. It also makes uncertainty explicit: decisions can be based on predictive quantiles or exceedance probabilities rather than on a point estimate whose reliability is unknown.

3. Digital Twin Architecture and Data Assimilation

3.1 Surrogate prediction with Gaussian processes

Repeated evaluation of a nonlinear finite-element or history-dependent confinement model can be costly when many load scenarios or calibration samples are required. A surrogate model is therefore used to approximate the mapping from input features z to performance outputs. The feature vector may include confinement ratio, normalized jacket stiffness, unconfined strength, load-path descriptors, current damage estimate, and maximum prior strain. A Gaussian-process surrogate is appropriate when data are sparse and uncertainty estimates are required, because it provides both a predictive mean and a predictive variance [13]. For a test feature z_, the predictive mean is

$$\mu(z_*) = m(z_*) + k_*\text{^}T\,(K + \sigma_n\text{^}2\,I)\text{^}-1\,(y - m)$$

and the predictive variance is

$$\sigma_*\text{^}2 = k(z_*, z_*) - k_*\text{^}T\,(K + \sigma_n\text{^}2\,I)\text{^}-1\,k_*$$

where m(z) is the prior mean function, k(·,·) is a positive-definite kernel, k_ is the vector of kernel values between z_ and training inputs, K is the training kernel matrix, σ_n^2 is observation noise variance, I is the identity matrix, and y is the vector of observed or simulated targets. The mean provides a fast prediction, while the variance indicates where additional simulation or monitoring evidence would be most informative. This is valuable in a digital twin because calibration experiments and sensor placement can be chosen to reduce predictive uncertainty rather than simply increasing data volume.

The surrogate is not intended to replace mechanics. Instead, it approximates a structured model over a bounded input domain. Extrapolation beyond the domain of confinement ratios, load amplitudes, or damage states represented in the calibration set must be flagged by the predictive variance and by domain-of-validity checks. A deep network could also approximate the mapping [19], but a Gaussian process provides a transparent uncertainty structure that is advantageous when data are limited and the prediction is used for assessment.

3.2 Bayesian calibration and state updating

Calibration updates mechanical parameters θ and surrogate hyperparameters φ using available evidence. The posterior distribution is

$$p(\theta, \varphi|Y) \propto p(Y|\theta, \varphi)p(\theta, \varphi)$$

where p(θ, φ) is the prior distribution and p(Y | θ, φ) is the likelihood of the observed history. The prior for θ should encode admissible ranges from confinement mechanics and design models, not arbitrary regularization. Bayesian calibration of computer models provides a formal way to combine simulation output with physical observations and to represent model inadequacy [14]. In the digital twin, calibration can occur offline from laboratory or field data and online as new monitoring data arrive. Online updating is particularly relevant for variable loading because the posterior after a cyclic sequence can differ from the posterior after monotonic loading even when the same specimen is considered.

The likelihood must reflect the measurement process. If measurement errors are approximately Gaussian and independent, a diagonal covariance may be adequate; if strain and displacement observations are correlated through the same deformation mode, a full covariance or a hierarchical error model is more appropriate. Model discrepancy should not be conflated with measurement noise. Systematic bias caused by neglecting jacket debonding or localized crushing belongs in a discrepancy term or in additional latent variables, whereas sensor noise belongs in the observation covariance. This separation is essential for credible extrapolation and for avoiding overconfident posterior distributions.

3.3 Sensitivity and baseline comparisons

Global sensitivity analysis identifies which inputs or calibration parameters dominate predictive variance. Variance-based sensitivity measures can allocate output uncertainty to confinement ratio, load-path descriptors, damage parameters, and measurement noise [15]. Such analysis is directly useful for test design: if predictive uncertainty is dominated by the rupture-strain parameter, additional hoop-strain instrumentation is more valuable than additional axial displacement measurements. It also prevents calibration from overfitting parameters that have little influence on the performance metric of interest.

Credible baselines are necessary for assessing whether the digital twin adds value. A deterministic confinement model, a purely data-driven regressor, and a tree-ensemble predictor [16] provide different reference points. The deterministic model tests whether mechanics alone is sufficient; the purely data-driven model tests whether the state-space structure is unnecessary; the ensemble provides a strong nonlinear baseline for tabular features. The digital twin should be judged not only by mean prediction error but also by calibration of uncertainty, behavior under load-path shift, and interpretability of updated states. These criteria are more demanding than a single aggregate error metric and are appropriate for a performance-prediction framework intended to support engineering decisions.

4. Variable Loading Prediction and Calibration

4.1 Load-path features and fabrication variability

Variable loading effects enter the twin through load-path descriptors and through their influence on damage accumulation. Useful descriptors include maximum compressive strain, cyclic amplitude, mean stress, number of significant cycles, unloading stiffness, and residual strain after unloading. These features are not merely additional regressors; they condition the state transition and modify the interpretation of measured strain. For example, a given hoop strain may imply different damage states depending on whether it occurs on first loading or after repeated unloading, because residual concrete dilatation and jacket-concrete interface degradation alter the confinement effectiveness.

Fabrication and installation quality also affect the mapping from design variables to actual confinement. Jacket thickness, overlap length, impregnation quality, alignment, and in-situ robotic placement can introduce geometric or material variability that is not represented by nominal design values. Process evaluation for automated robotic fabrication of in-situ confinement indicates that fabrication workflow and quality control are relevant to the as-built confinement system [17]. In the digital twin, such factors should enter as covariates or uncertain parameters when inspection data are available; otherwise they contribute to model discrepancy. This use of fabrication information is subsidiary to the mechanics-based prediction problem, but it prevents the twin from treating nominal jacket properties as exact.

4.2 Damage accumulation and residual capacity

Damage under variable loading is represented as an irreversible state variable driven by strain history. A flexible formulation is

$$D(t) = 1 - exp[-\int_0^t (|\dot{\varepsilon}_c(\tau)|/\varepsilon_u)^m \, d\tau]$$

where D(t) is damage at time t, $\dot{\varepsilon}_c(\tau)$ is the axial strain rate, ε_u is a reference ultimate strain, and m is a positive exponent controlling sensitivity to strain amplitude and rate. This expression is a proposed history-dependent damage law, not an empirically fitted result. It has the desirable properties that damage is nonnegative, increases with accumulated strain, and remains bounded below one. The exponent m can be treated as an uncertain parameter and calibrated against cyclic or variable-amplitude tests. When m is large, damage is concentrated in high-strain excursions; when m is small, moderate cyclic strains contribute more strongly to degradation.

Residual capacity can then be expressed as a function of initial capacity and damage. A simple admissible form is

$$R(t) = R_0\big(1 - D(t)\big)^q$$

where R_0 is the initial capacity and q is a deterioration exponent. This relationship is not a substitute for a full failure criterion; it provides a monotone link between accumulated damage and residual performance that can be updated as evidence accumulates. For design assessment, R(t) can be compared with demand S(t), and the predictive distribution of R(t) − S(t) can be used to estimate exceedance probabilities. The value of q and the mapping from D(t) to specific limit states must be established through calibration and mechanistic reasoning rather than assumed to be universal.

4.3 Calibration design and uncertainty propagation

Calibration design should target parameters that are both influential and identifiable. If only axial force and displacement are measured, the individual contributions of jacket modulus and hoop strain may be difficult to separate, because both influence lateral pressure. Adding hoop-strain measurements or local jacket deformation measurements improves identifiability. Bayesian optimization can select additional load cycles or measurement locations that maximize expected information gain while respecting testing constraints [18]. This is particularly relevant for variable loading, where the most informative experiment may be a sequence of unloading-reloading cycles rather than a single monotonic test.

Uncertainty propagation proceeds from prior to posterior and then to prediction. Process noise w_k accounts for unmodeled mechanics, parameter uncertainty is represented by the posterior of θ, surrogate uncertainty is represented by the Gaussian-process variance, and measurement noise is represented by the observation covariance. These sources should be propagated jointly rather than added as independent error terms after prediction. The resulting predictive distribution supports reliability-based decisions, including comparison with serviceability limits and determination of inspection intervals. The twin is therefore a decision-support model, not an autonomous authority; engineering judgment and code-based verification remain necessary.

5. Technical Discussion and Boundary Conditions

5.1 Model discrepancy and identifiability

The principal technical risk in a digital twin for FRP-confined concrete is confusing data-driven fit with mechanical validity. A flexible surrogate can reproduce calibration data while learning a physically inconsistent mapping, especially if load-path descriptors are correlated with specimen-specific artifacts. The state-space formulation reduces this risk by constraining the evolution of strain, confinement, and damage, but it does not eliminate it. Model discrepancy must be represented explicitly when systematic deviations are observed. Physics-based inverse modeling and model validation are therefore central to digital twins, not peripheral activities [20].

Identifiability is a second boundary condition. Parameters that are not excited by the available loading history cannot be estimated reliably. A monotonic test may identify initial stiffness and peak capacity but provide weak information on cyclic damage parameters. Conversely, a severe cyclic test may identify damage evolution but confound residual capacity with prior damage. Calibration protocols should therefore be designed around the decision target. Verification and validation frameworks for digital twins emphasize that the intended use determines the required fidelity and evidence [21]. This principle applies directly here: a twin used for serviceability screening needs different evidence from one used for ultimate-capacity prognosis.

5.2 Comparison with alternative prediction strategies

A purely empirical strength model is simpler and may perform well within its training domain, but it cannot represent load-path-dependent state or update from monitoring data in a physically interpretable way. A finite-element model with detailed concrete plasticity and interface behavior can represent more mechanisms, but it is computationally expensive and still requires calibration of uncertain parameters. The proposed twin occupies an intermediate position: it uses mechanics to structure the state transition, a surrogate to reduce computational cost, and Bayesian updating to assimilate evidence. This combination is

appropriate when repeated prediction under multiple load scenarios is required and when uncertainty must be communicated.

Alternative confinement models could be embedded in the virtual model. For example, stress-strain formulations that explicitly represent the interaction between concrete dilatation and jacket stiffness provide a different parameterization from design-oriented models [22]. The digital twin architecture is model-agnostic in the sense that the transition operator F can be replaced or augmented, provided that the state variables and uncertainty structure remain coherent. This modularity is an advantage, but it also means that the twin's credibility depends on the chosen constitutive model and on transparent documentation of its assumptions.

5.3 Design and lifecycle implications

For engineering practice, the framework suggests that performance prediction under variable loading should be expressed as a probabilistic trajectory rather than a single capacity check. Monitoring data can update the trajectory, and the updated state can inform inspection, repair, or load restriction decisions. The twin can also support post-event assessment by comparing observed strain histories with predicted damage states. These uses require that the model remain connected to physical variables: jacket strain, confinement pressure, concrete strain, and damage. A prediction system that produces only a scalar utilization ratio without an internal state may be adequate for screening but is poorly suited to lifecycle decision support.

The framework also clarifies data requirements. Reliable prediction under variable loading benefits from hoop-strain measurements, load history records, and periodic stiffness estimates. Fabrication and installation records improve the mapping from design to as-built confinement. Calibration data should include variable-amplitude and cyclic loading, not only monotonic tests, because the damage parameters are weakly informed by monotonic response. These requirements follow from the structure of the model and from established observations of cyclic confinement behavior [6], not from a claim that a particular monitoring system has already been deployed.

6. Conclusion

This paper has formulated a digital twin-assisted framework for performance prediction of FRP-confined concrete structures under variable loading. The framework combines a mechanics-based virtual model of lateral confinement, a state-space representation of strain, confinement, damage, and load-history effects, a Gaussian-process surrogate for efficient prediction, and Bayesian calibration for evidence-based updating. The equations developed for confining pressure, state transition, observation, predictive mean and variance, posterior calibration, damage accumulation, and residual capacity provide a coherent analytical basis for the twin. The approach addresses a specific gap: monotonic confinement models and purely empirical predictors do not adequately represent history-dependent performance, while detailed finite-element models are difficult to update repeatedly under sparse monitoring data.

The main technical implication is that prediction should be organized around latent physical states and explicit uncertainty rather than around a single fitted strength value. This structure allows load-path features to influence damage evolution, permits inspection and fabrication information to enter as covariates or discrepancy terms, and supports reliability-oriented decisions. The discussion also identifies boundary conditions: parameter identifiability depends on the loading and measurement design; model discrepancy must be separated from sensor noise; and surrogate extrapolation must be constrained by the domain

of calibration. These conditions are not incidental limitations but define the evidence required for credible use.

The framework is methodological and does not assert field-validated predictive accuracy. Its value lies in providing a disciplined architecture for integrating confinement mechanics, variable-load damage, surrogate modeling, and Bayesian updating. Before operational deployment, the damage and residual-capacity relationships require calibration against variable-amplitude and cyclic tests, and the twin must be verified against independent structural response data. When developed with those safeguards, a digital twin can support condition-aware assessment of FRP-confined concrete structures and make the transition from static design prediction to lifecycle performance prediction technically defensible.

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